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SUMMARY

~~FINAL~~ REPORT - Part A.

December 1961

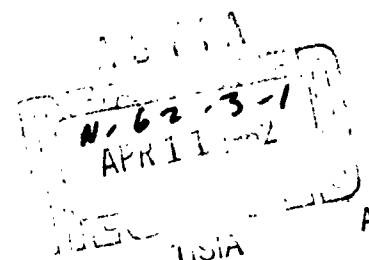
CONTRACT : AF 61 (052)- 328.

OBJECTIVE OF THE WORK : Study of electronic traps in dielectrics
by thermoluminescence.

AUTHORS : G.Bonfiglioli, P.Brovetto, C.Cortese.

TITLE : " Apparatus for Thermoluminescence ".

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~~Σ~~
SUMMARY: The detailed description is given of an apparatus for thermoluminescence (above room temperature i.e. up to about 350°C), having the following peculiar features: the linear rate of warming is obtained through a cam - actuated Variac transformer ; the light detection makes use of phase rectification; the apparatus is provided with a special furnace for recording "glow current curves". The possibility of rendering "flat" the spectral response of the photomultiplier ~~between 2000 Å and 6000 Å~~ through a composite filter is investigated and the method of calculating such a filter is developed. The performances of the apparatus ~~look quite~~ ^{were} satisfactory , ~~in spite of the rather simple and relatively unexpensive construction of each part.~~

Thermoluminescence is becoming a method of increasing interest for the investigation of certain electronic properties of dielectrics - as is shown by the growing amount of literature on this subject. (1)

The technical and experimental problems with which one is faced when using this method are by no means trivial and a very careful design is necessary to obtain results significant and

(1) For a bibliography on the subject spoken of which is rather extensive although not complete, the reader is referred to several Technical Notes by the same Authors , issued under ARDC Contracts No. AF 61(514)-1333 , No. AF 61(052)-328 and to the Proceedings of the International Conference on Color Centers and Crystal Luminescence , Torino , Sept. 1960, published by the Istituto Elettrotecnico Nazionale Galileo Ferraris.

free from errors which are often difficult to detect.⁽²⁾

On the other hand, detailed descriptions of thermoluminescence equipments are lacking since usually several delicate points are not fully explained. Last not least, the amount of electronics usually employed both in piloting the linear rate furnace and in light detecting is relatively complicated and expensive.

This paper describes an apparatus which features some not quite usual design, achieving very good performances at a moderate degree of complication and cost.

A block scheme of it is shown in Fig. 1 :

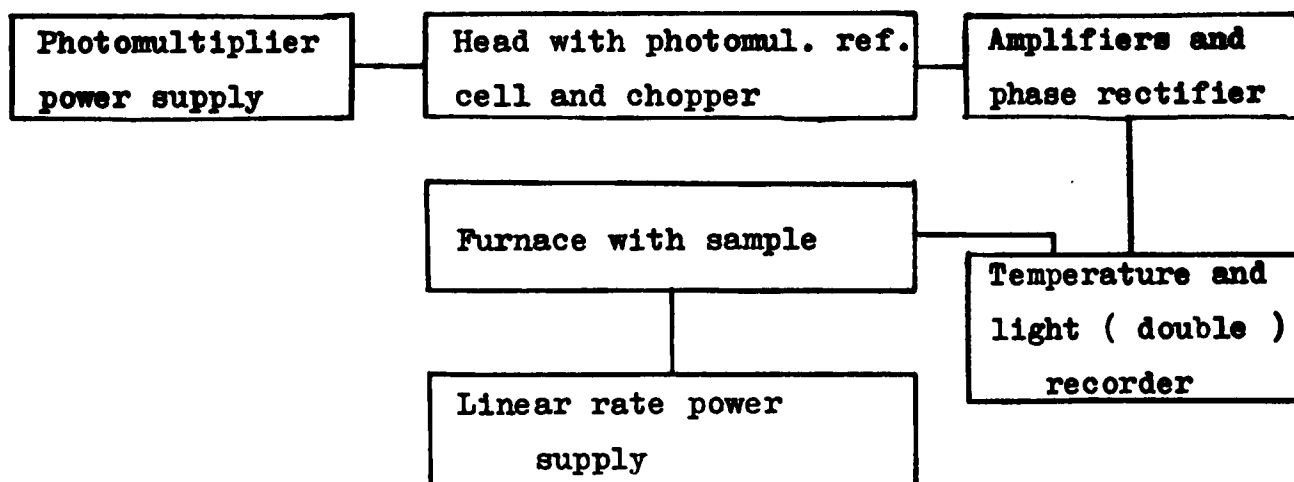


Fig. 1.

A general view of the apparatus is shown in Fig. 2.

(2) A striking example of such a type of error is stressed in D.R. Lewis, T.N. Whitaker, C.W.Chapman, Ann. Mineral. 44,1221 (1959) at the pg. 1127.

A)- FURNACE AND ITS SUPPLY.

The furnace has been so designed as to achieve a high degree of temperature uniformity of the specimen.

The measurement of the (instantaneous) temperature is performed by a thermocouple located within the light collecting pipe, which is capable of giving very accurately the temperature of the emitting surface of the sample (which is usually a single crystal of five to thirty cubic millimeters size).

Fig. 3 shows the details of the construction.

The maximum power of the furnace is around 150 Watt, as requested for a maximum temperature in the neighbourhood of 350°C. Fig. 4 shows the circuit which supplies the power to the furnace and in Appendix A are given the calculations of the cam used to achieve a linear rate of warming.

It is to be emphasized that this method was chosen not as much for economy reasons, as rather because glow curves are very sensible to the derivative of the warming speed. In fact it was our feeling that electronic regulation systems are likely to give control of the average speed of warming, perhaps very accurate but not sufficiently "smooth".

Fig.5 shows the record of a typical linear run of our apparatus, still uncorrected for the thermocouple response.

The degree of linearity achieved (once corrections for the non linearity of the chromel-alumel element are allowed for is of $\pm 1\%$ within the whole range of temperature covered in a run.

Different speeds of warming are possible (down to 0.2°C/sec) changing the small motor and the cam. The temperature is recorded on a channel of a conventional two pens recorder (Siemens mod. T22P7-79), which incorporates a transistor amplifier (full scale

deviation : 100 mm. corresponding to 6 mV. A shunt was used to extend this range to about 15 mV).

A') - THE FURNACE WITH ELECTRIC FIELD.

Fig. 6 shows a second furnace, to be used in conjunction with the same equipment spoken of and to be supplied following the same method described above. It is to be employed when recording so called "glow current curves", that is curves of electrical conductivity vs (linearly growing) temperature.

Some references of investigations performed through glow current curves are listed in footnote⁽³⁾.

This furnace has been so designed as to fulfill the following requirements: good uniformity both of temperature and of electric field inside the specimen; possibility of simultaneous light detection; very high insulation, resisting to high voltages (up to 15 KV) with negligible leakage currents.

Fig. 6 clearly illustrates how the above requirements have actually been met in present design.

B) - THE PHOTOMULTIPLIER HEAD.

Fig. 7 clearly shows the construction of the head which locates the photomultiplier, the reference Selenium pho-

(3) P.Dutton,R.Maurer, Phys.Rev. 90, 126 (1953)

G.Chiarotti,N.Inchauspé , Phys.Rev. 109, 345 (1958)

A.A.Braner, A.Halperin, M.Sieger,Proc. Int.Conf. on Color Centers and Crystal Luminescence, Torino, Sept. 1960; published by the Istituto Elettrotecnico Nazionale, pg. 46 ff.

tocell , its (24 V, 0.12 A) illuminating lamp, the rotating chopper and the synchronous motor driving the latter.

A series of different photomultipliers was used according to the particular problem, as for example: a DuMont 6291 , 10 stages, S 11 response for visible spectrum and medium-low sensitivity; or a DuMont K 1306 , S 13 response and quartz window for UV; or a DuMont 6911 for infrared - according to the problem dealt with. But, when possible, an RCA 6810 A (14 dynods, S 11 response) was used , to achieve the highest possible sensitivity.

The photomultiplier is supplied by a conventional stabilized power supply capable of delivering 15 mA between 400 and 3000 V , with a stability degree of $\pm 3 \cdot 10^{-4}$ against network voltage fluctuations up to $\pm 10\%$. The two outputs of the photomultiplier and of the Selenium cell are fed to separate amplifiers, followed by a phase rectifier. About this last point, we must remark that the rectified output obviously depends upon the relative phase of the signal and the reference input. This phase can be varied within fairly large limits by mechanically displacing the reference photocell along the periphery of the chopper disc , untill the maximum rectified output is obtained . The details of the construction can be seen on Fig. 7.

C) - THE CIRCUITRY.

Fig. 8 shows the complete scheme of the circuitry. The preamplifier and cathode follower unit (gain 50), located within the photomultiplier head, is used only with low gain phototubes , and it is disconnected when 6810 A tube is operating.

The chopper light signal is then fed to the amplifier , which consists of two conventional RC coupled stages , giving a

gain of 30. The reference signal voltage, on the other hand, is amplified through a resonant capacitor-transformer circuit and then fed to a cathode follower, resulting in a voltage of 2 X 15 V peak to peak.

After phase rectification , the rectified output goes to a vacuum tube voltmeter, which in turn drives the second channel of the (Siemens) recorder.

The phase rectification unit is based on a well known bridge circuit with two non-linear elements on two opposite arms (actually, Ge diodes, OA85). The principles of its operation can be found in the literature⁽⁴⁾, and can be substantially summarized in the fact that the zero frequency beat between the two synchronous signals (of the photomultiplier and the Selenium cell) is made use of. One big advantage obviously consists of the fact that only the chopped frequency (actually 240 C/sec) signals are detected , cutting off in a drastic way all the disturbing components of different frequency due, say, to phototube noise, ac ripple etc. But, maybe the most important feature offered by this method is that it is particularly suited to give an output strictly linear vs. the amplitude of the input signal, no matter whether very small or rather large input voltages are involved. This is a point which cannot be overemphasized in T.L. experiments , where very often accurate ratios are wanted of the height (or areas) of glow peaks of widely different magnitude.

(4) L.I. Farren, Wireless Eng. 23, 330 (1946)

R.H. Dishington, Proc. IRE 37, 1401 (1949)

S. Krishnan, Electronic and Radio Eng. 36, 45 (1959)

S. Krishnan, R. Chidambaram, Electrical and Radio Eng., 36, 314 (1959)

D) - " FLAT " RESPONSE OPTICAL FILTER.

A further point which has been considered is related to the circumstance that the light emitted by the sample during the growth of its temperature has a varying wavelenght.

In particular, the wavelenghts belonging to various glow peaks are generally not the same. Since the current response of the phototube is not uniform vs. the wavelenght of the incoming light , this obviously implies that the areas subtended by two recorded glow peaks are not in the same ratio as the number of luminophor centers which have recombined with emission of photons. Sometimes the wavelenghts of these photons are known from independent measurements⁽⁵⁾, and in this case, once the response of the photocathode is also known, it is possible to perform the correction. But very often these data are lacking, and the analysis of the glow curves is severely handicapped.

This is the reason which pushed us to consider the problem of building a composite filter capable of "flattening" the response of the phototube over a substantially wide range of wavelenghts.

This seems actually possible, with the sacrifice of roughly a factor of 3 in sensitivity, using commercial phototubes and filters. Since the construction of such a filter has not yet been carried out, we limit ourselves to derive, in Appendix B , a method suitable for its calculation, letting to a future Report the description of the experimental results.

(5) Cfr. for example: G.Bonfiglioli, P.Brovetto, C.Cortese, Phys. Rev. 104, 951 and 956 (1959).

It is to be remarked at this point that, to use such a filter, requires that the sample be a point source of light. This compelled us to keep rather large the distance between the sample and the phototube, with some sacrifice in sensitivity.

E) - PERFORMANCES.

The performances of the apparatus have proved satisfactory. Fig.10 shows the overall linearity of the setup, by a curve whose ordinates are the recorded output signals while the abscissas represent the amount of light(white light from a tungsten filament lamp) reaching the cathode of the phototube.⁽⁶⁾

It can be seen that a rough linearity is obtained up to rather large signals, say, $2/3$ of the full scale deviation of the Siemens recorder (1,6 V) . Fig. 11 shows another curve having the same meaning as the previous one, since it is actually nothing else than the initial portion of the curve of Fig.10 obtained with a

(6) A simple device, well visible in Fig. 9 has been used to obtain different amounts of light whose respective ratios (not the absolute values) are known very accurately. It consists of a source of light equipped with a screen carrying a number of holes, which can be left open or alternatively can be obtured with screws. When, say, two holes are left open, the passing light is the sum of the lights passing when one or the other hole is open. Using a certain number of such combinations and measuring the corresponding outputs, one can easily obtain a graphical representation input/output like that of Fig. 11 - without the necessity of an absolute calibration of the source of light.

different recorder of higher sensitivity.⁽⁷⁾ The full scale deviation of Fig. 11 corresponds to 1/2 the full scale of Fig. 10.

The linearity within this restricted range is now satisfactory.

The reduction achieved in the noise level is evidenced in Fig. 12, where the oscillogram shows the superposition of the noise to a signal picked up before the rectification stage, while the pen-recorded curve represents the corresponding dc output.

The gain in signal to noise ratio is clearly very great. Fig. 13 finally shows as an example the record of a glow curve, from a NaCl sample irradiated with X rays, and scanned at about 0.2 °C/sec.

The sensitivity of the thermoluminescence apparatus is obviously limited by the residual level of noise after rectification - which depends in turn on the phototube used, its temperature (room temperature in present setup), its operating voltage, and so on.

To quote some definite figure we can say that a crystalline sample of NaCl of 70 mg., irradiated with roughly 10^{16} F-centers/cm³, and scanned at the lowest speed of 0.2 °C/sec, furnishes

(7) This is a Graphispot from Séfram(Paris). It is a combination of a sensitive mirror galvanometer equipped with variable calibrated shunt, with a split photocell servomechanism operated by the galvanometer spot and carrying recorder pen. In this way high accuracy (0.5%), high sensitivity (up to 1 mV for 250 mm. deviation) records can be obtained of the phenomenon investigated. The fidelity is also high, since the system is able to respond with sensible deviations to signals varying with time at much faster rates than actual T.L. signals.

a glow peak at about 400 °C (it is the biggest one) whose amplitude is about 100 times the noise. From this figure the limit of sensitivity can be estimated in a way suitable for practical use.

It can finally be observed that in our setup the distance between the sample and the photocathode is rather large (60 cm) and could be reduced if necessary where would a higher sensitivity be required. One of the reasons of this considerable distance has been mentioned in paragraph D.

APPENDIX A - Project of the cam.

Let us consider a thermal capacity C of a material with an infinite thermal conductivity. This is in fact the approximate assumption under which the present calculation has been carried out, and which besides experiment has proved to give a largely sufficient accuracy.

If a thermal flux linearly growing vs time is generated within our thermal capacity, say:

$$\phi(t) = \phi_0 (1 + at) \quad (1)$$

the temperature T will be ruled by the following equation:

$$C \frac{dT}{dt} = \phi - K(T - T_0) \quad (2)$$

where K stands for the external thermal conductivity, and T_0 for the ambient temperature. Through eq.(1),eq.(2) becomes:

$$\frac{dT}{dt} = \frac{\phi_0}{C} (1 + at) - \frac{K}{C} (T - T_0) \quad (3)$$

It is very easy to verify that :

$$T = T_0 + \beta t \quad (4)$$

is a solution of these eq. (3), provided:

$$\phi_0 = C\beta \quad (5) \quad ; \quad a\phi_0 = K\beta \quad (6)$$

Let us remark that the flux $\phi(t)$ is now given by

$$\phi(t) = \beta (C + Kt) \quad (7)$$

so that a sudden variation of the main supply voltage, multiply-

ing by a constant factor the thermal flux, produces the same effect on the warming rate β .

To obtain (experimentally) the values of C and K, which must be known in order to calculate the profile of the cam, it is sufficient to remember that eq.(3) tells that the cooling law of the thermal capacity C from an initial temperature T_1 down is given by:

$$T = T_0 + (T_1 - T_0) e^{-\frac{K}{C} t} \quad (8)$$

The ratio K/C can therefore be measured rather easily and with sufficient accuracy, whilst C can be evaluated at least approximately through the weights and the chemical composition of the furnace.

Clear enough, to obtain, through electrical resistors, a law like (7) , we shall make use of a voltage having a constant part, as well as an increasing one. Being

$$\phi(t) = \frac{V^2(t)}{R} \quad (9)$$

by the Ohm's law,(where R stands for the resistance of the furnace wiring, assumed constant), it follows:

$$V(t) = \sqrt{R\beta C} \sqrt{1 + \frac{K}{C} t} \quad (10)$$

The initial voltage being:

$$V_0 = \sqrt{R\beta C} \quad (11)$$

eq. (10) becomes:

$$V(t) = V_0 \sqrt{1 + \frac{K}{C} t} \quad (12)$$

Such a result can be obtained through the circuit of Fig. 14 provided the law of the movement of the second Variac transformer is suitably chosen.

V_m , the maximum voltage, depends on the maximum temperature desired T_m through the equation:

$$V_m = V_0 \sqrt{1 + \frac{K}{C} \frac{T_m}{\beta}} \quad (13)$$

Along the sliding contact of the second Variac transformer the voltage should vary according to the law:

$$V(t) = V_0 \left(\sqrt{1 + \frac{K}{C} t} - 1 \right) \quad (14)$$

therefore the angle $\theta(t)$ of rotation of the same Variac is given by the equation:

$$\theta(t) = \frac{V(t)}{V_m - V_0} \theta_m = \theta_m \frac{V_0}{V_m - V_0} \left[\sqrt{1 + \frac{K}{C} t} - 1 \right] \quad (15)$$

where θ_m stands for the maximum allowable angle (somewhat less than 2π , of course).

To pilot the rotation, we used a cam and a flexible steel wire, coupled to each other as Fig. 14 shows.

Under these conditions we must have, if r stands for

the radius of the drum connected to the Variac transformer:

$$r \frac{d\theta}{dt} = \rho(\varphi) \omega \quad (16)$$

if ω stands for the (constant) angular speed $d\varphi/dt$ of the motor which drives the cam. This equation and (15) give, for the searched function $\rho(\varphi)$:

$$\rho(\varphi) = \frac{B}{\sqrt{1 + A\varphi}} \quad (17)$$

that is to say:

$$\rho(\varphi) = \frac{\rho_0}{\sqrt{1 + \frac{A}{\omega} \varphi}} \quad (18)$$

where we have put:

$$\begin{cases} A = \frac{K}{C} \\ B = r \theta_m V_0 \frac{K}{2 C \omega (V_m - V_0)} \end{cases} \quad (19)$$

(20)

ρ_0 , the initial radius of the cam, can usefully be connected to the condition that the cam moves, for example, $3/4$ of 360° , while the Variac transformer rotates of the angle θ_m . That is:

$$\rho_0 \int_0^{3\pi/2} \frac{d\varphi}{\sqrt{1 + \frac{A}{\omega} \varphi}} = r \theta_m \quad (21)$$

which yields:

$$\rho_o = \frac{r \theta_m}{\frac{2\omega}{A} \left(\sqrt{1 + \frac{A}{\omega} \frac{3\pi}{2}} - 1 \right)} \quad (22)$$

Putting the right member of this equation equal to the right member of eq.(18) and using eq. (20), we obtain finally:

$$\omega = \frac{3\pi}{2} \frac{\beta}{T_m} \quad (23)$$

which determines the angular speed of the motor.

Resuming ,given the furnace, that is C,K,R the wanted warming rate β , the maximum temperature T_m , the initial temperature T_o , the maximum allowable rotation of the second Variac θ_m , and the radius r of the Variac drum , we get:

$$\begin{aligned} V_o &= \sqrt{R\beta C} \\ V_m &= V_o \sqrt{1 + \frac{K}{\theta} \frac{T_m}{\beta}} \\ \rho_o &= \frac{r \theta_m T_m K}{3\pi \beta C \left(\sqrt{1 + \frac{K}{C}} - 1 \right)} \\ \rho(\varphi) &= \rho_o / \sqrt{1 + \frac{K}{C} \frac{\varphi}{\omega}} \\ \omega &= \frac{3\pi}{2} \beta / T_m \end{aligned}$$

which solve the problem completely.

APPENDIX B - Calculation of the flat response optical filter

Let us assume that on the surface element $dx dy$ of the photocathode impinge $N_\lambda dx dy$ photons of wavelength λ . The corresponding elementary photocurrent will be $j(\lambda, x, y) dx dy$.

Let us assume further that the total area A of the photocathode be subdivided in partial regions of areas a_i , each covered with a filter of transmittancy $\tau_i(\lambda)$, being further agreed that $\tau_0(\lambda) \equiv 1$, meaning that the surface a_0 is not covered by any filter. If it occurs that two filters, $\tau_r(\lambda)$ and $\tau_s(\lambda)$ overlap (we exclude overlapping of more than two filters) the overlapped area will be labelled a_{rs} .

For sake of simplicity, from now on, we consider in detail the case of two filters only, whereby:

$$a_0 + a_1 + a_2 + a_{12} = A \quad (1)$$

If now a light of spectral distribution $I(\lambda)$ (evaluated in number of photons) impinges on the photocathode, screened by the filters as just explained, the photocurrent coming from the element $dx dy$ belonging to the area a_1 will be:

$$j(\lambda, x, y) dx dy = \frac{I(\lambda) \tau_1(\lambda)}{N_\lambda}$$

Integrating on the whole area, we get the total current:

$$i(\lambda) = \frac{I(\lambda)}{N_\lambda} \left\{ \iint_{a_0} j dx dy + \tau_1(\lambda) \iint_{a_1} j dx dy + \right.$$

$$+ \tau_2(\lambda) \iint_{a_2} j \, dx dy + \tau_1(\lambda) \tau_2(\lambda) \iint_{a_{12}} j \, dx dy \} \quad (2)$$

Now, our problem can be stated as follows: with a given number of filters(2 in our case) and a given number of regions of the cathode (4, in our case, a free one, a_0 , two covered by one filter each, a_1 , and a_2 and one covered by both, a_{12}), to determine the values of the four areas(of which 3 only are independent, as prescribed by eq.(1)) in such a way as to obtain a current i approximately independent of λ , that is to say, we want:

$$X = \frac{i(\lambda)}{I(\lambda) N_\lambda}$$

to be a quantity approximately constant vs λ .

The explicit expression of X is:

$$X = \iint_{a_0} j \, da + \tau_1 \iint_{a_1} j \, da_1 + \tau_2 \iint_{a_2} j \, da_2 + \tau_1 \tau_2 \iint_{a_{12}} j \, da_{12} \quad (3)$$

To solve our problem, something must be known about the function $j(\lambda, x, y)$ and we will make the position:

$$j(\lambda, x, y) = \varphi(\lambda) \rho(x, y) \quad (4)$$

This corresponds to the assumption that the spectral response of the photocathode is uniform on its whole area, whilst the sensitivity varies from a point to another(the central region being usually the most sensitive).

From eq.s(1) and(4) , eq. (3) can be written:

$$X = \varphi(\lambda) \left[\iint_A \rho \, dA - (1 - \tau_1) \iint_{a_1} \rho \, da_1 - \right. \\ \left. - (1 - \tau_2) \iint_{a_2} \rho \, da_2 - (1 - \tau_1 \tau_2) \iint_{a_{12}} \rho \, da_{12} \right] \quad (5)$$

Since the first integral is actually a constant- let us call it M - through the following positions.

$$\frac{X}{M} = \zeta \quad ; \quad \frac{\iint_{a_1} \rho \, da_1}{M} = \eta_1 \\ \frac{\iint_{a_2} \rho \, da_2}{M} = \eta_2 \quad ; \quad \frac{\iint_{a_{12}} \rho \, da_{12}}{M} = \eta_{12} \quad (6)$$

we obtain:

$$\zeta = \varphi(\lambda) \left[1 - (1 - \tau_1) \eta_1 - (1 - \tau_2) \eta_2 - (1 - \tau_1 \tau_2) \eta_{12} \right] \quad (7)$$

This equation will be used to find the values of $\eta_1, \eta_2, \eta_{12}$ and then, being $\rho(x, y)$ a known function, the values of the areas $a_1, a_2, a_{12} - a_0$ being deduced from eq. (1). Eq. (7) is of the type:

$$\zeta = F(\lambda; \eta_1; \eta_2; \eta_{12}) \quad (8)$$

where the dependence on $\eta_1, \eta_2, \eta_{12}$ is linear.

There are several ways of making use of eq.(7). The simplest one is to choose 4 wavelenghts, say, $\lambda_\alpha, \lambda_\beta, \lambda_\gamma, \lambda_\delta$ and to put:

$$\begin{aligned}
 F(\lambda_a; \eta_1, \eta_2, \eta_{12}) &= F(\lambda_\beta; \eta_1, \eta_2, \eta_{12}) \\
 F(\lambda_\beta; \eta_1, \eta_2, \eta_{12}) &= F(\lambda_\gamma; \eta_1, \eta_2, \eta_{12}) \quad (9) \\
 F(\lambda_\gamma; \eta_1, \eta_2, \eta_{12}) &= F(\lambda_\delta; \eta_1, \eta_2, \eta_{12})
 \end{aligned}$$

Eq.s(9) form a system of linear non omogeneous equations in η_1 , η_2 , η_{12} and then ,given the arbitrariness of choice of λ_a , λ_β , λ_γ , λ_δ , they are certainly compatible , since the determinant of the coefficients can always be made non vanishing.

Another way, which can furnish a better solution, stands on the application of the method of least squares.

Let us consider n wavelenghts λ_i ($i = 1, 2, 3, \dots, n$) and the squared differences

$$\Delta_i = \left[\xi - F(\lambda_i; \eta_1, \eta_2, \eta_{12}) \right]^2 \quad (10)$$

We shall assume as the "best" values of the unknowns those for which $\sum_i \Delta_i$ is a minimum. This criterion yields:

$$\sum_i \left[F(\lambda_i; \eta_1, \eta_2, \eta_{12}) - \xi \right] \frac{\partial F(\lambda_i)}{\partial \eta_1} = 0 \quad (11 a)$$

$$\sum_i \left[F(\lambda_i; \eta_1, \eta_2, \eta_{12}) - \xi \right] \frac{\partial F(\lambda_i)}{\partial \eta_2} = 0 \quad (11 b)$$

$$\sum_i \left[F(\lambda_i; \eta_1, \eta_2, \eta_{12}) - \xi \right] \frac{\partial F(\lambda_i)}{\partial \eta_{12}} = 0 \quad (11 c)$$

$$\sum_i \left[F(\lambda_i; \eta_1, \eta_2, \eta_{12}) - \xi \right] = 0 \quad (11 d)$$

Eq.n. (11a) prescribes that the sum of the squares of the differences should be a minimum also vs. ξ . In fact, the "best" obtainable minimum can be got letting unknown also the value of ξ and imposing the "best" constancy of the function F in the aforementioned sense.

Eq.n. (11 a) can be written explicitly:

$$\sum_{i=1}^n \left\{ \varphi(\lambda_i) \left[1 - (1 - \tau_1)\eta_1 - (1 - \tau_2)\eta_2 - (1 - \tau_1 \tau_2)\eta_{12} \right] - \xi \right\}$$

$$\varphi(\lambda_i) (1 - \tau_1) = 0 \quad (12)$$

that is :

$$\begin{aligned} & \left\{ \sum_{i=1}^n \varphi^2(\lambda_i) (1 - \tau_1)^2 \right\} \eta_1 + \left\{ \sum_{i=1}^n \varphi^2(\lambda_i) (1 - \tau_1)(1 - \tau_2) \right\} \eta_2 + \\ & + \left\{ \sum_{i=1}^n \varphi^2(1 - \tau_1 \tau_2) (1 - \tau_1) \right\} \eta_{12} + \left\{ \sum_{i=1}^n \varphi(\lambda_i) (1 - \tau_1) \right\} \xi = \\ & = \sum_{i=1}^n \varphi^2(\lambda_i) (1 - \tau_1) \end{aligned} \quad (13a)$$

Eq.s. (11 b), and (11 d) lead to equations perfectly analogous to eq.n(13 a) - and can be written at once, what we will not make for sake of brevity.

Now, if $\rho(x,y) = \text{const.}$, it would be straightforward to deduce from the values of the η_i the values of the partial areas a_i . Otherwise, the evaluation of the a_i depends upon the actual form of the function ρ and upon the geometry of the problem, which is to be fixed a priori through practical consideration. A geometry

which has been considered for our experiments is shown in Fig.15.

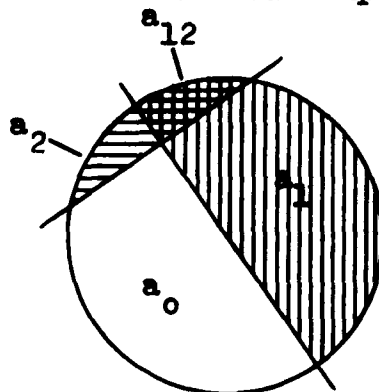


Fig. 15.

and a reasonable form for $\rho(x,y)$ can be for example:

$$\rho = C \exp \left[- a (x^2 + y^2) \right]$$

The detailed calculation relative to this choice will be given together with the experimental results , when the application to a practical case will be reported.

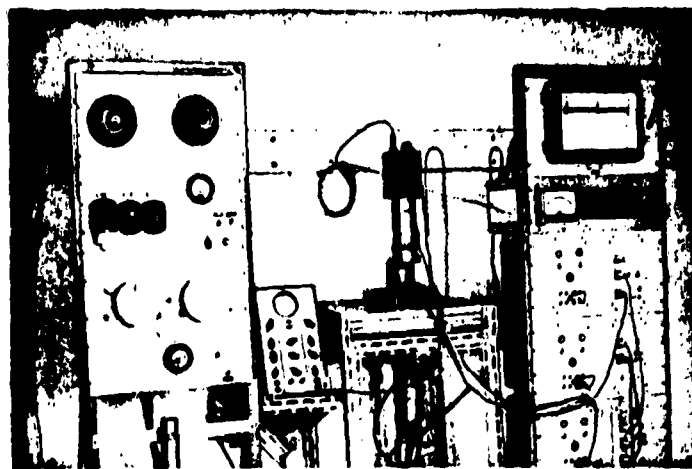


fig. 2



fig. 7

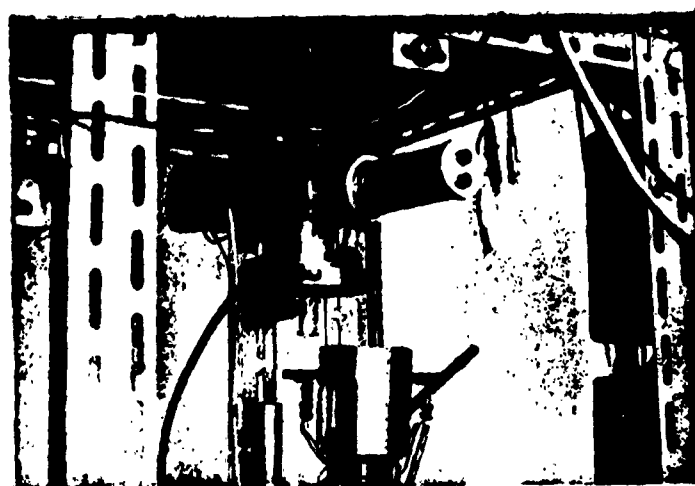
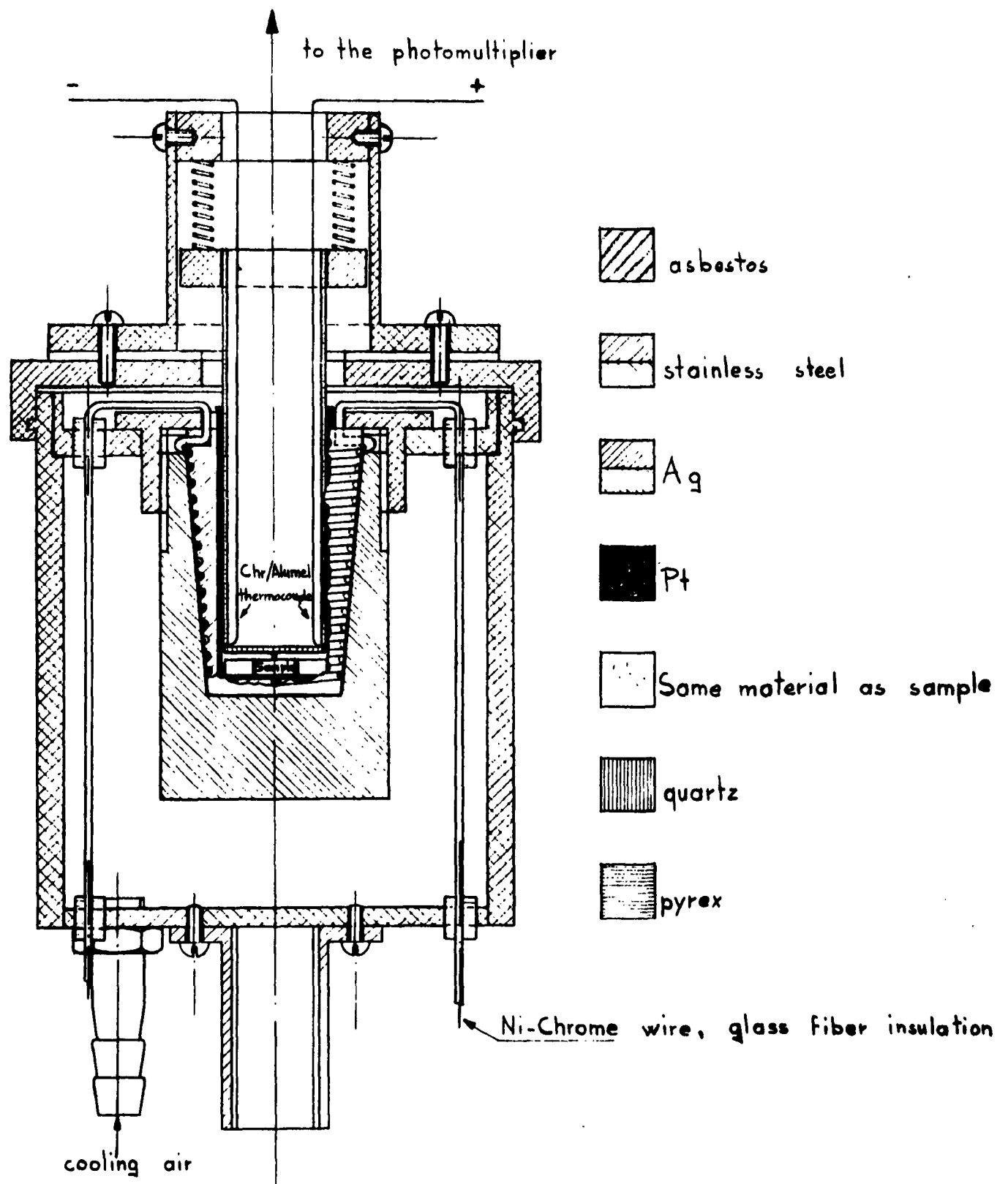


fig. 9



Thermoluminescence Furnace

Fig. 3

scale 1:1

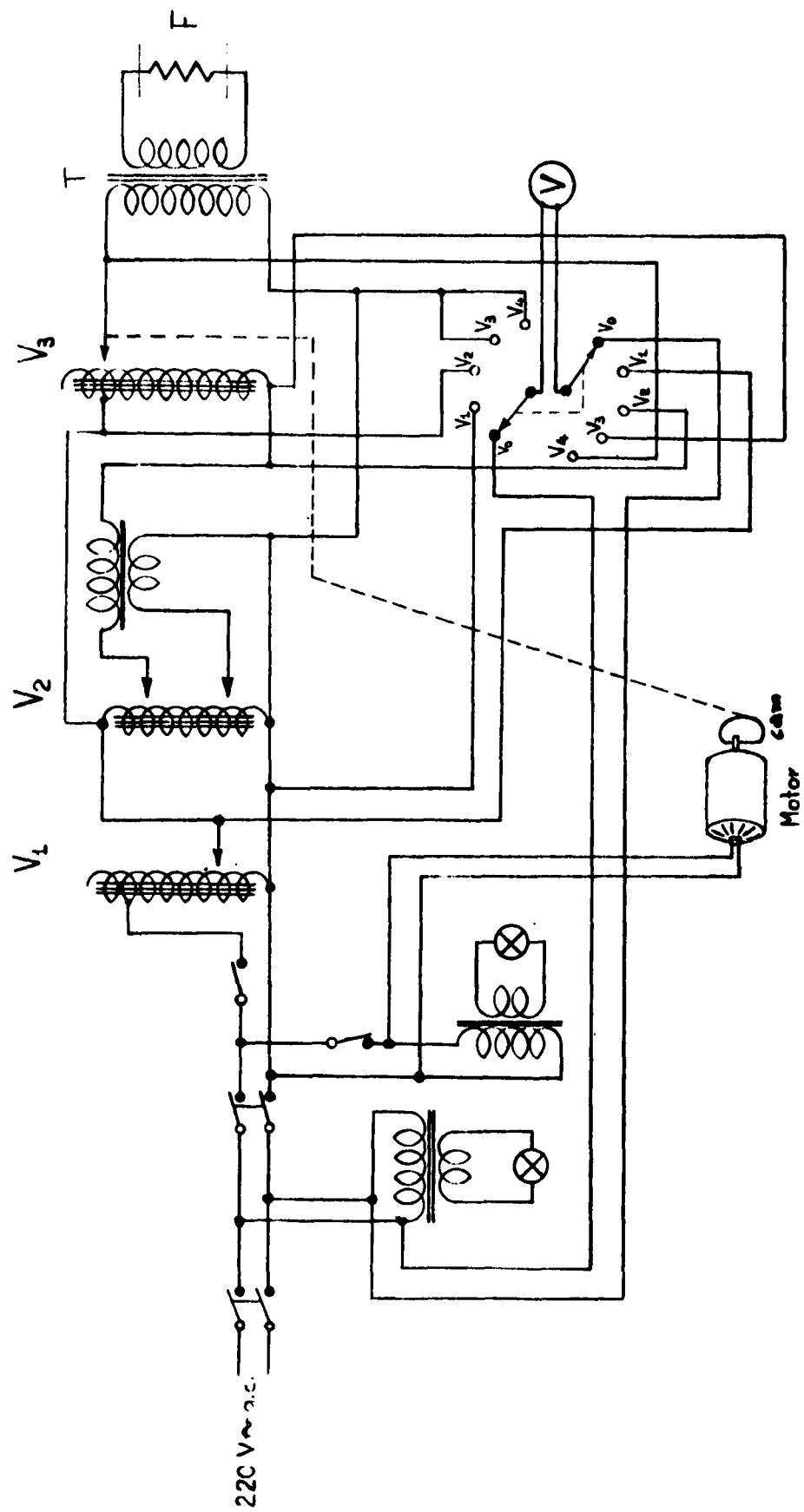


Fig. 4

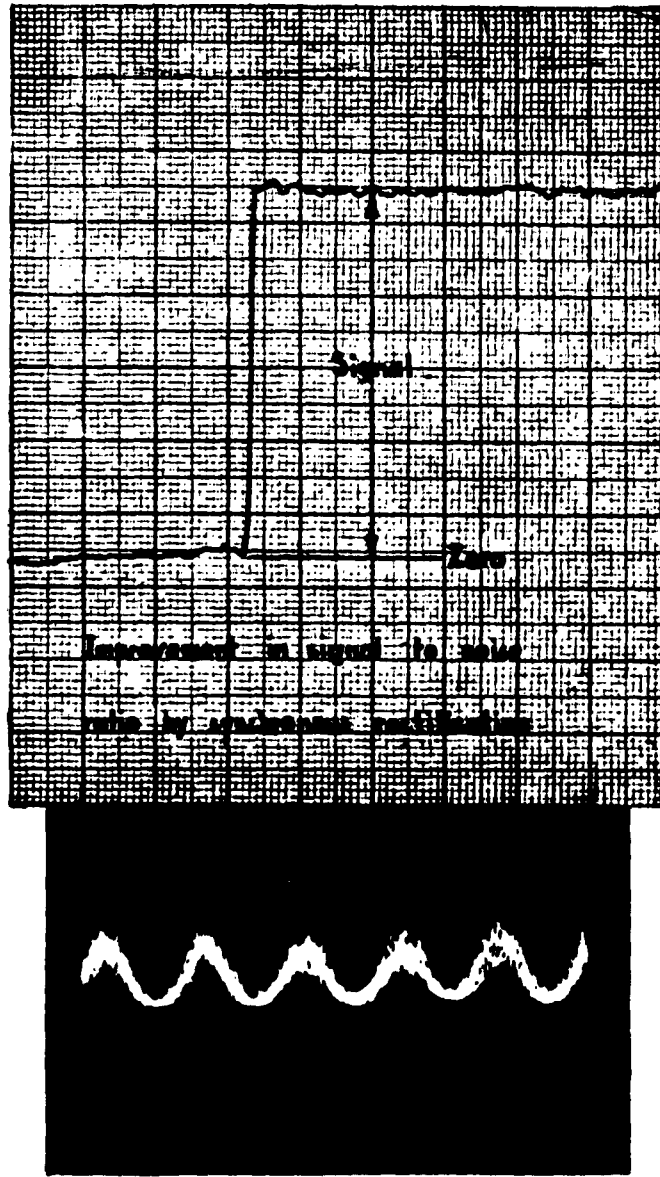


fig. 12

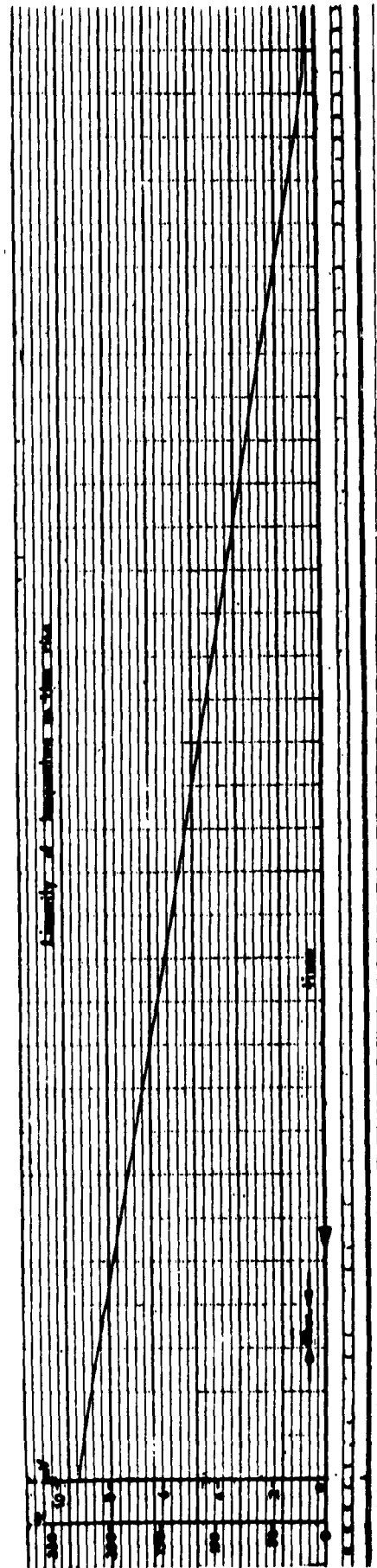


Fig. 5

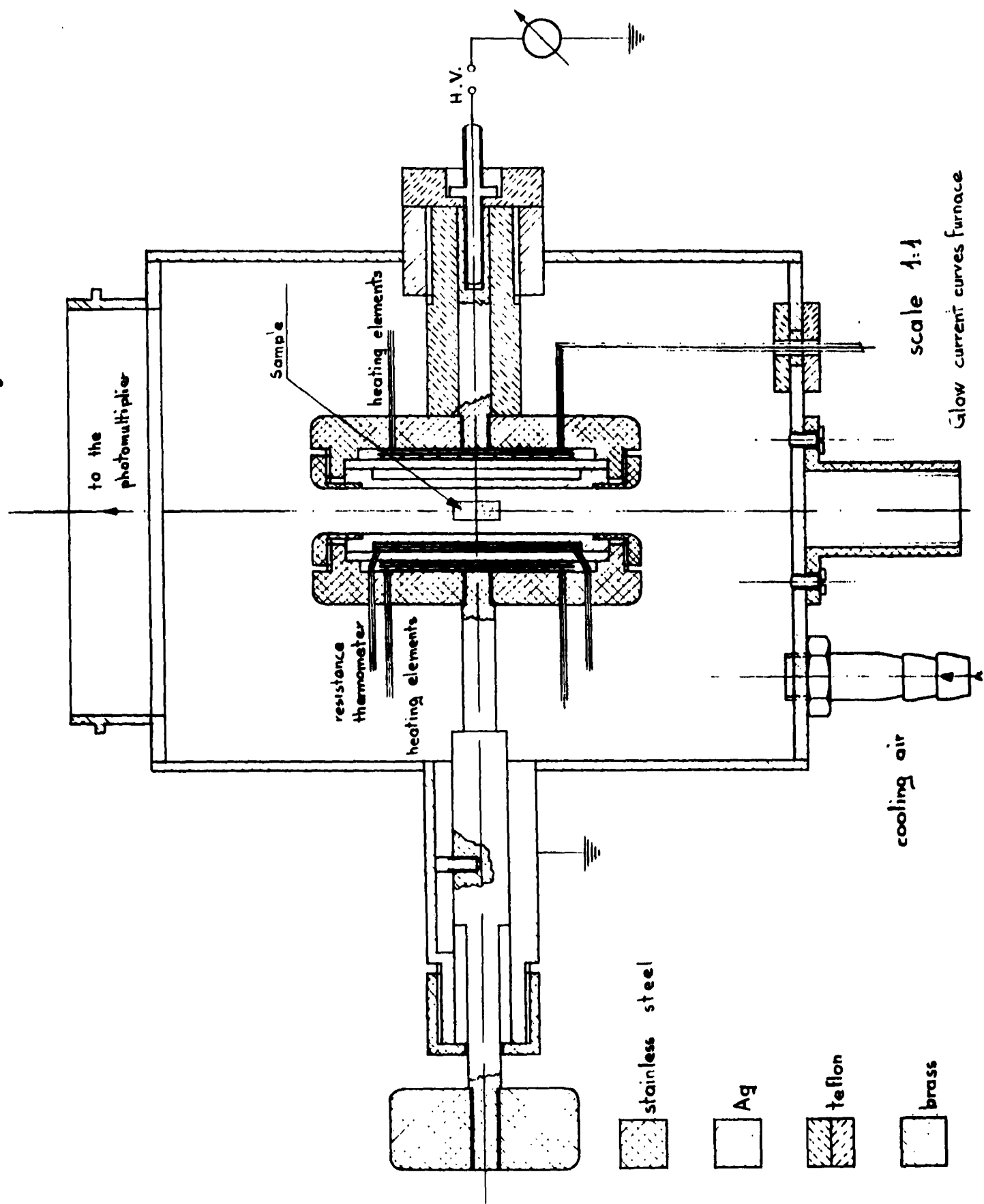


Fig 6

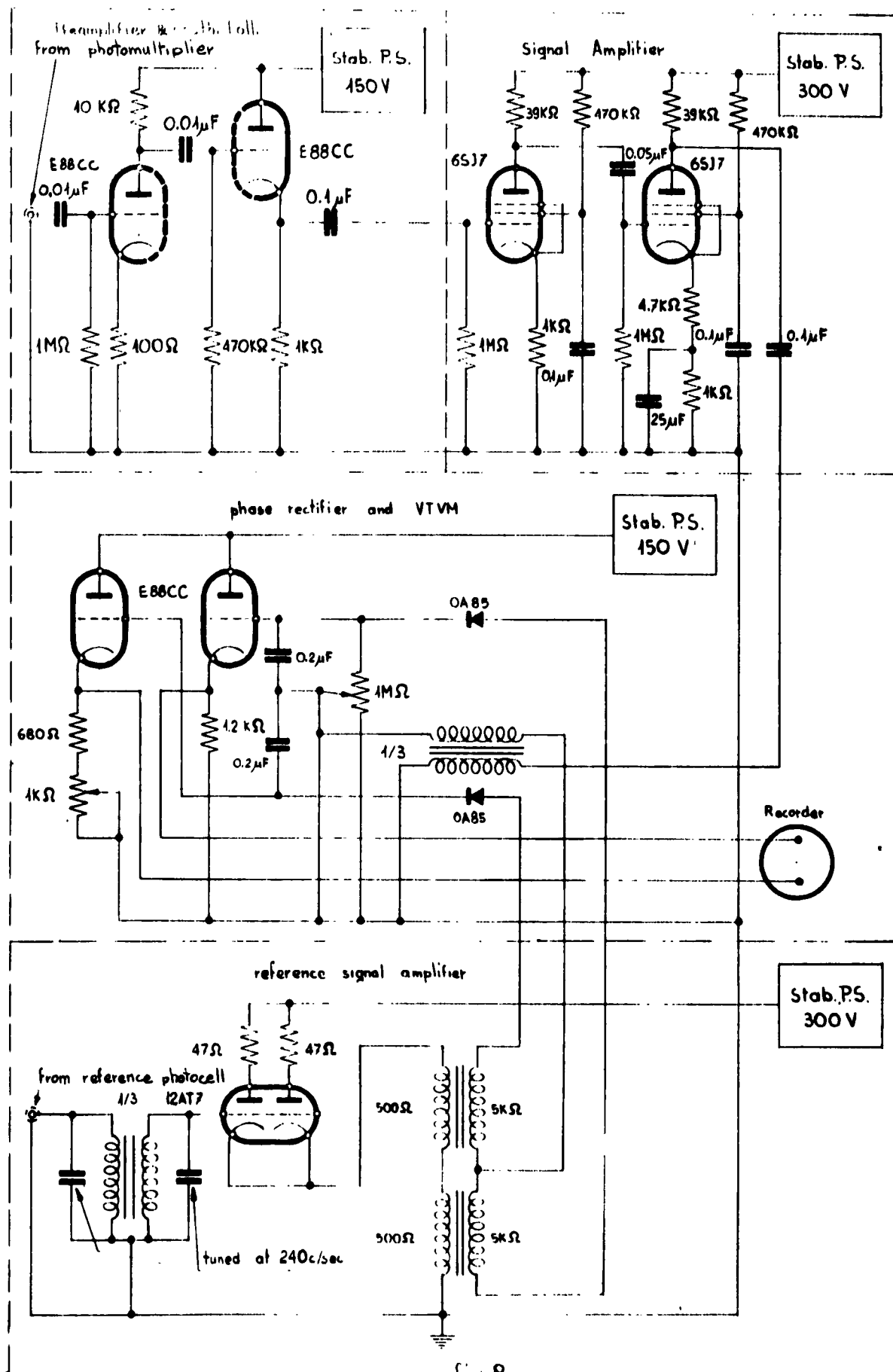


Fig. 8 -

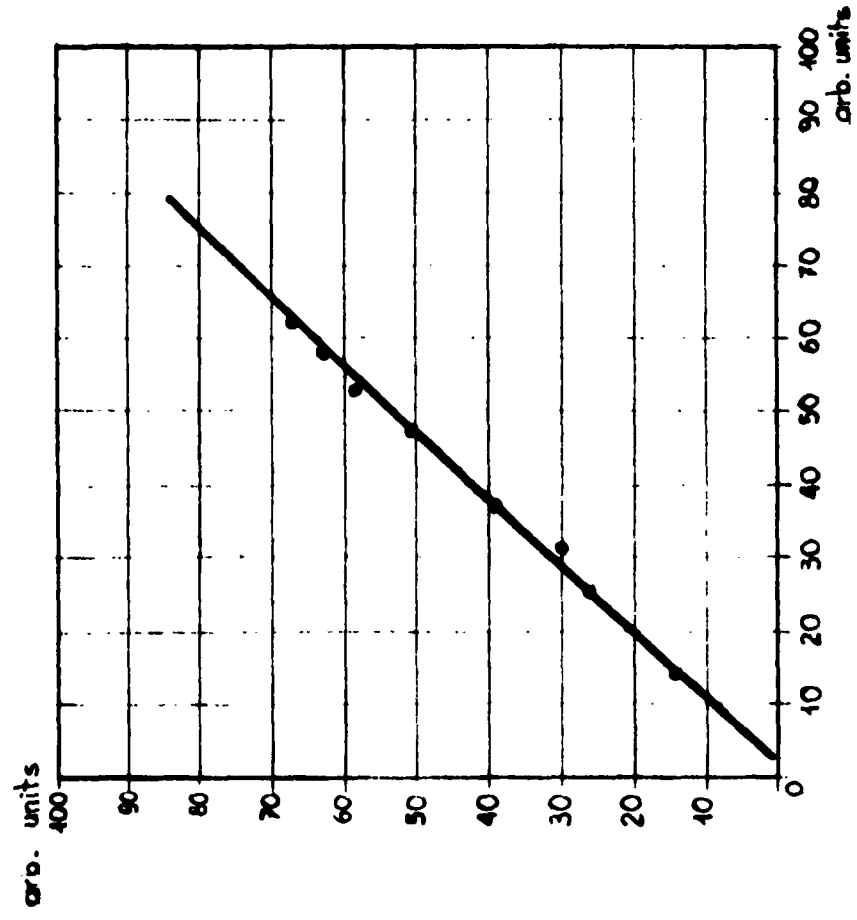
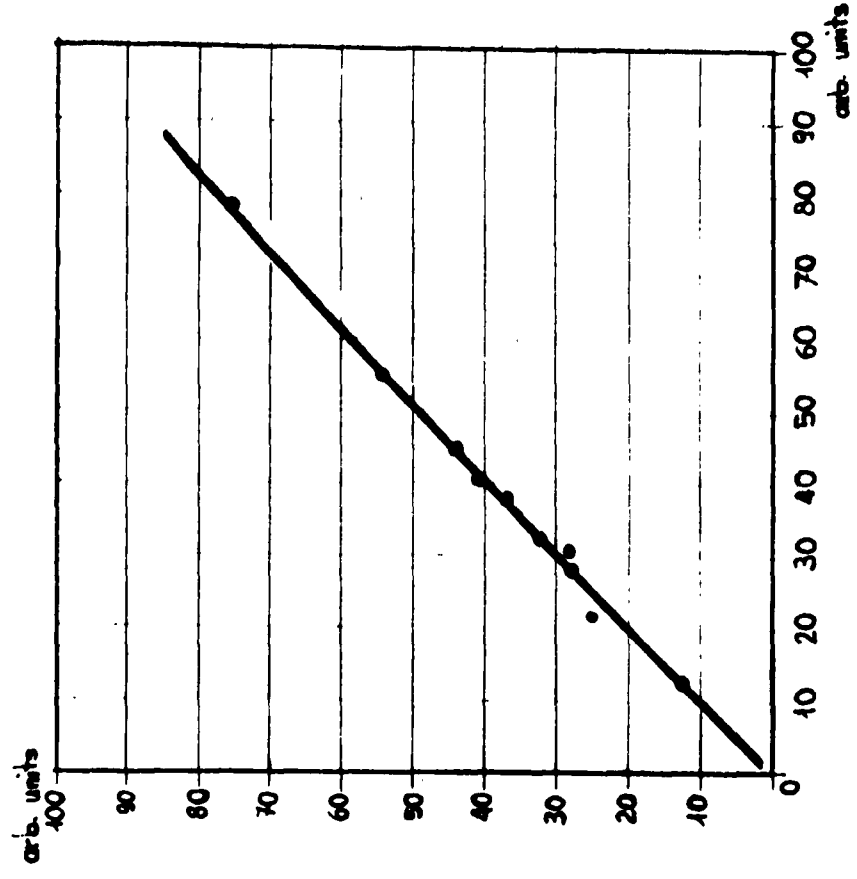


Fig. 10



overall linearity of the apparatus

Fig. 11

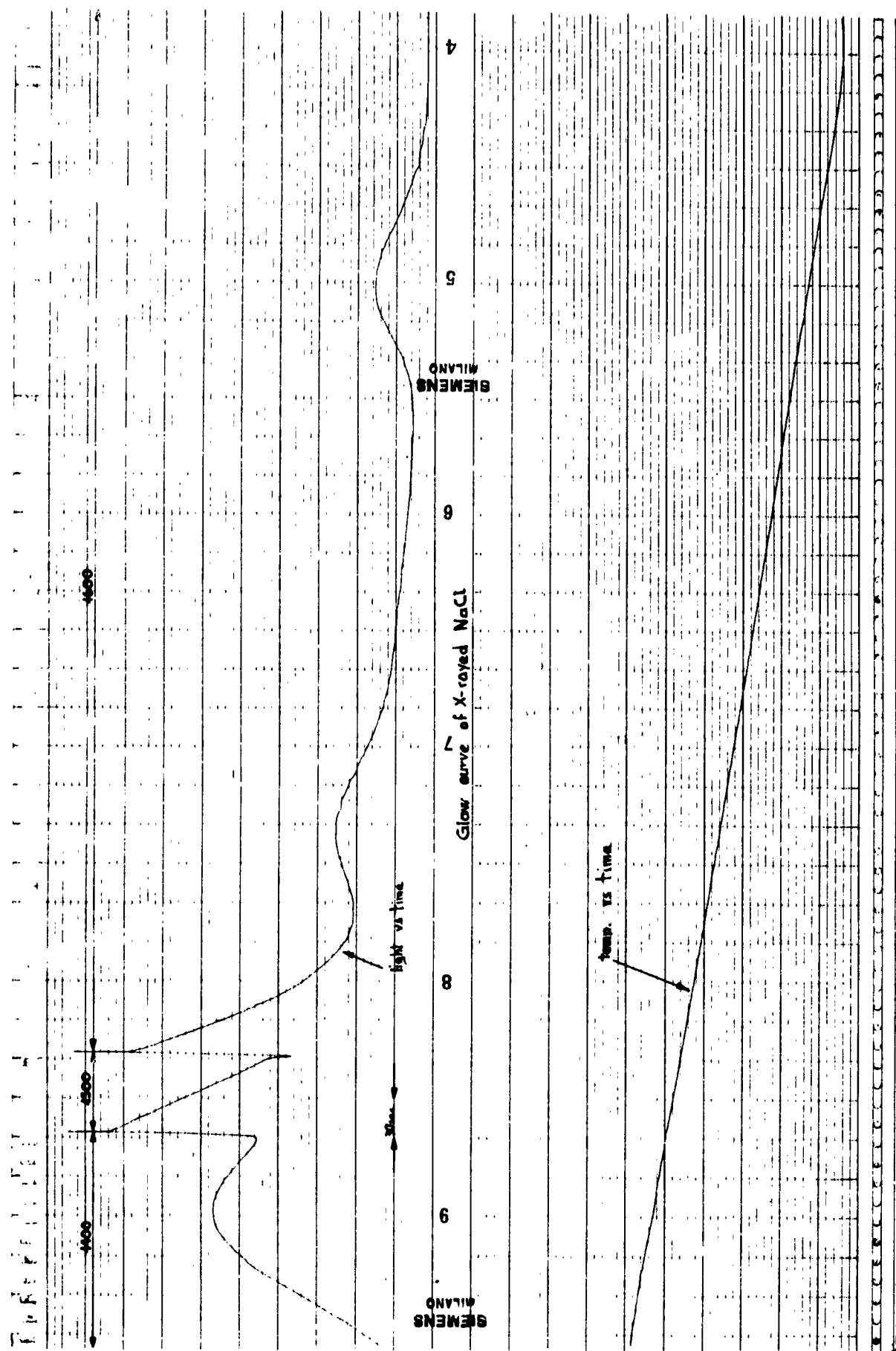


fig 13

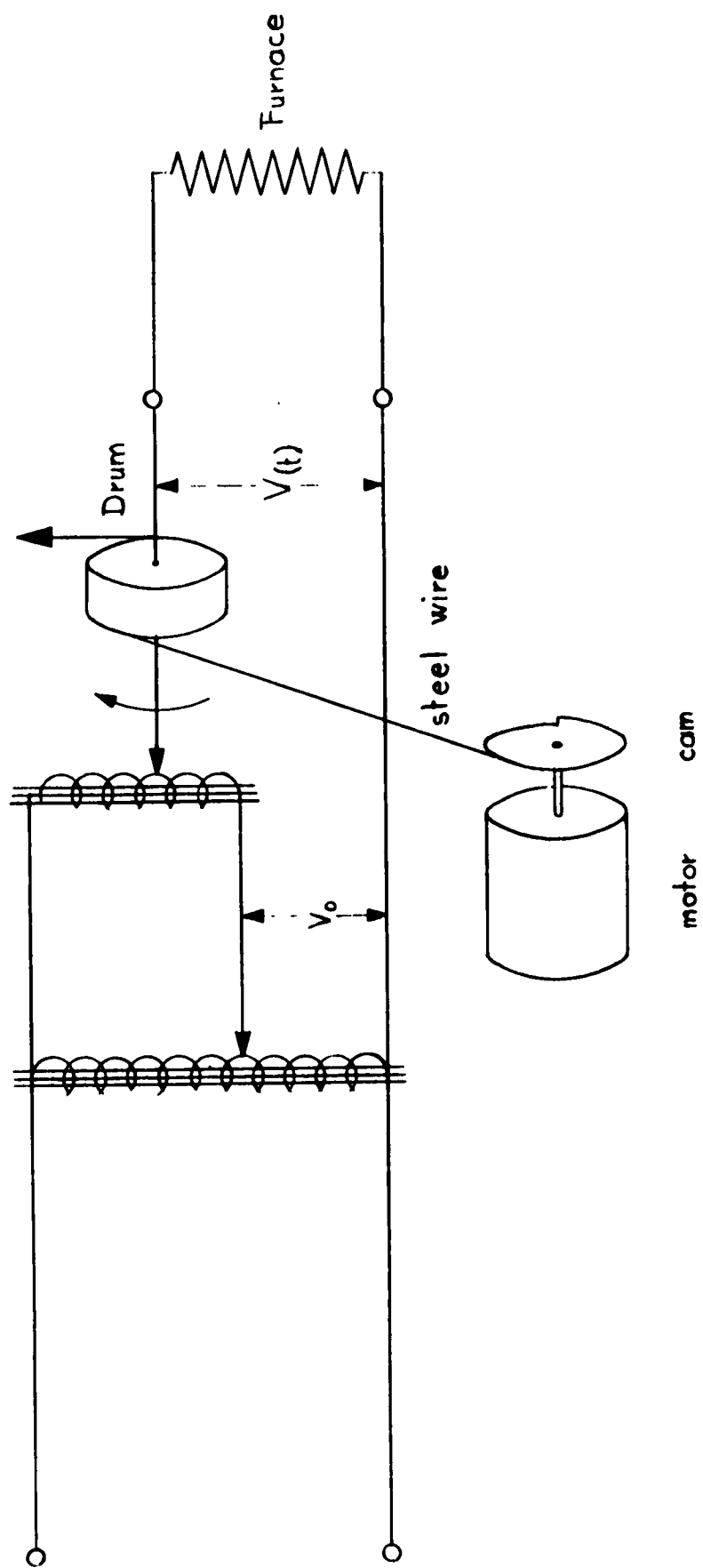


Fig. 14